

Cambridge International AS & A Level

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

October/November 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1 Solve the equation $2^{3x-1} = 5(3^{1-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$ where a and b are integers. [4]

[illegible]

- 2 The polynomial $2x^3 - x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $(2x + 3)$ is a factor of $p(x)$.

(a) Find the value of a .

[2]

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(b) When a has this value, solve the inequality $p(x) < 0$.

[4]

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- 3** The equation of a curve is $y = \sin x \sin 2x$. The curve has a stationary point in the interval $0 < x < \frac{1}{2}\pi$.
Find the x -coordinate of this point, giving your answer correct to 3 significant figures. [6]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4 (a) Express $4 \cos x - \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [3]

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- (b) Hence solve the equation $4 \cos 2x - \sin 2x = 3$ for $0^\circ < x < 180^\circ$. [5]

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- 5 (a) Solve the equation $z^2 - 6iz - 12 = 0$, giving the answers in the form $x + iy$, where x and y are real and exact. [3]

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- (b) On a sketch of an Argand diagram with origin O , show points A and B representing the roots of the equation in part (a). [1]

(c) Find the exact modulus and argument of each root.

[3]

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(d) Hence show that the triangle OAB is equilateral.

[1]

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6 Relative to the origin O , the points A , B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}.$$

(a) Using a scalar product, find the cosine of angle BAC .

[4]

[illegible]

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- 7 The variables x and θ satisfy the differential equation

$$x \sin^2 \theta \frac{dx}{d\theta} = \tan^2 \theta - 2 \cot \theta,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 2$ when $\theta = \frac{1}{4}\pi$.

- (a) Show that $\frac{d}{d\theta}(\cot^2 \theta) = -\frac{2 \cot \theta}{\sin^2 \theta}$.

(You may assume without proof that the derivative of $\cot \theta$ with respect to θ is $-\operatorname{cosec}^2 \theta$.) [1]

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- (b) Solve the differential equation and find the value of x when $\theta = \frac{1}{6}\pi$. [7]

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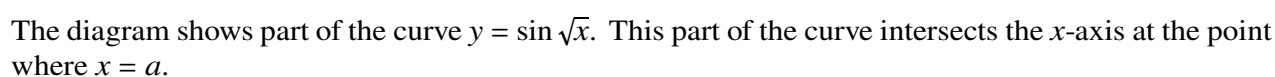
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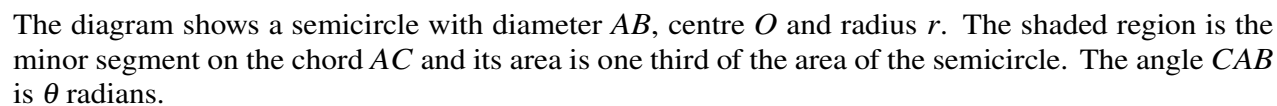
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- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) Verify by calculation that $0.5 < \theta < 0.7$.

[2]

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- (c) Use an iterative formula based on the equation in part (a) to determine θ correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

[3]

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10 Let $f(x) = \frac{4 - x + x^2}{(1 + x)(2 + x^2)}$.

(a) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (b) Find the exact value of $\int_0^4 f(x) \, dx$. Give your answer as a single logarithm. [5]

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This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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